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To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

VOL. 4 BATON ROUGE, LA., JAN.-FEB.-MARCH, 1930. NO. 5

THE CLEVELAND MEETING

High levels of mathematical interest in this part of the Southland were reached when in the little city of Cleveland, Mississippi, on March 7, 8, were carried out with most remarkable effectiveness, joint programs of the Louisiana Mississippi Section of the Mathematical Association of America, the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics, and the Mississippi Academy of Sciences. It is to the lasting credit of Cleveland, the Delta State Teachers College, the local Rotarians, the local Chamber of Commerce and the officers of the Council and the Section, that this two-day convention of three bodies was put over in so admirable a manner.

Deserving special commendation were the labors and service of Miss Julia Dale upon whom had fallen the double office of the secretaryship of the Section and leadership, in conjunction with Chairman Hardin and President Kethley, in the maturing of all needed preparations for the triple occasion.

—*S. T. S.*

AN UNSURPASSED ARRANGEMENT

And now what have we in this Mississippi-Louisiana land? We have two distinct but mutually interested bi-State mathematical organizations under mutual contract to hold their respective annual meetings jointly, that is, at the same time and in the same city. One year such meetings are in one of the two States, the next year, in the other of the two. In the year in which the Section-Council meeting is held in Louisiana a third body meets conjointly with them, namely, the Louisiana Academy of Sciences. When Section and Council meet in Mississippi, in joint meeting with them is the Mississippi Academy of Sciences.

—*S. T. S.*

THE NEW OFFICIAL FAMILIES

Attention of readers and subscribers is directed to the title page of the News Letter on which are the names of the recently elected officers of the Section and the Council. With both Chairman Nichols and Chairman Tucker thoroughly committed to the Letter we may reasonably look forward to a banner year, not only in respect to our journal, but in respect to every department of the two organizations' activities. —*S. T. S.*

PERSONALITY IN MATHEMATICS TEACHING

Once we heard a colleague remark that "we are all impressionists", the "we" referring particularly to teachers. If we admit its truth, mathematics teachers are no exception to the principle. But a surplus of personality in a student no more makes for the assignment to him of higher grades by the mathematical instructor than an alert personality of the instructor makes for a heightened interest in mathematics on the part of the student. Indeed, in our judgment, the influence of personality in the instructor himself operates in far greater degree on the student than does that of the student in securing a high rating of his work.

While, personality, like the point, or the line, or the surface, is not easily defined, we are yet profoundly conscious of its basic value when in its presence.

In the mathematics classroom its manifestations are unmistakable. Unlimited delight in the subject matter; a zeal for arousing, when possible, a kindred pleasure in it on the student's part; human, natural reactions of the mind to the discoveries, surprises and problems that rise up along the path of mathematical inquiry; a burning devotion to the truth; an unconcealed scorn for pretense to knowledge when there is no knowledge—THESE are a few of the indubitable manifestations of PERSONALITY surcharged with the consciousness of a mission to save the world for mathematics. —*S.T.S.*

INTERESTING HIGH SCHOOL FRESHMEN IN ALGEBRA

By LUCILLE M. BOSTICK
Wright High School, New Orleans, La.

What is the emotion with which the average high school freshmen approach the study of algebra? The strange or unfamiliar content of the subject-matter excites in them curiosity, self-consciousness, and a certain amount of fear—the three elements that constitute the emotion of awe. This is a fruitful attitude, but whether they learn to like or dislike the subject depends largely upon the teacher.

What must be the qualifications of the teacher, other than mastery of the subject, who is to make the most of his attitude on the part of his pupils? First of all, he must love his subject, for if he is to make algebra attractive, he must put his heart and soul into it. In other words, he must be a good salesman, for genuine interest induces interest in others. Next, he must be sympathetic, letting his pupils realize that he is, at all times, their friend and helper, that he appreciates their difficulties and is ready to help them to succeed. Then, too, he must be a keen student of human nature—he must know and understand psychology. The curiosity of his pupils must be converted into interest and energy, the self-consciousness and fear changed to self-assertion and confidence.

How can this be accomplished? This is the problem of each individual teacher, and perhaps, a different problem with each class. However, some few general suggestions may be offered.

1. Go slowly at first, giving time for thorough understanding and appreciation of each step. The inexperienced teacher, only, fears that he will fail to cover the assignment if he does this. Speed can be acquired later. A thorough understanding is most important. No huge structure can ever be erected on a weak foundation. We do not want our pupils to stop with algebra. It is our great desire that their interest carry them on into geometry and higher mathematics. Therefore let them first master the fundamentals.

2. Try to make the work as simple as possible in the beginning. "Nothing succeeds like success," and confidence is three-fourths of success. Show wherein algebra is very much like arithmetic, only just a little shorter and perhaps a little bit easier. If we, as teachers, take that attitude our pupils will soon adopt it, and realize for themselves that algebra is not so new, nor so strange after all. Soon the element of fear will disappear altogether, and confidence take its place. Then, and then only, are they ready to tackle difficult situations, but if we once let them get discouraged the cause is lost, or at least the fear element becomes a greater problem with which to struggle.

3. Present each new step as attractively as is possible without sarificing the practical. Simple little devices help to fix facts, and sometimes reach the slower pupils who are not ready to grasp the abstract. Remember that they have not altogether outgrown the concrete presentations of their grammar school days just because they have attained high school age, and it becomes our problem to bridge this gap.

4. Permit the weaker pupils to spend more than one period a day in the algebra classroom, when possible. Where a teacher has several classes of Course I a day, this can be easily managed. The pupils have their regular lesson with their classmates first, and then, with the principal's permission, return during some study period and occupy desks in the back of the room where they take no active part in the lesson, but follow once more that instruction and, for practice, work some additional examples along the same line. In this way they often get a better understanding of that which they failed to comprehend during the first presentation. This is one way of keeping all pupils up with the class, and reduces considerably the percent of failures at the end of the term.

5. Give short assignments of home work in Course I, and subject matter that has already been worked up in class. This enables the pupils to do the work without help at home, and consequently trains them to work more independently. All new material should be worked in the classroom under the supervision of the teacher, where he can give helpful guidance and encouragement when needed. Often a well chosen question by the teacher helps to clear up a difficulty.

The attitude towards the subject acquired by the pupils is almost as important in Course I, as is the mastery of the subject matter. Therefore do every thing in your power to give your pupils a chance to become as interested in algebra as you are.

FINDING A PUPIL'S DIFFICULTIES IN ALGEBRA

By JOHN C. STONE
State Teachers College, Montclair, N. J.

The correct answer to a test question does not always indicate that a pupil understands the mathematical principle involved. The answer may be a matter of memory, or a mere manipulative skill. But it is the errors that a pupil makes that gives the teacher a chance to analyze and diagnose his difficulties. In order to find some of the difficulties that a pupil has, and thus to determine what phases of high school algebra need greater stress and better teaching, I gave the following test to 330 high school graduates selected from over 100 different high schools. All of these pupils had taken one year of algebra, and nearly half of them had two years. Moreover, they all ranked in the upper half of their graduating classes, and many were honor pupils.

An attempt was made to test their understanding of algebra rather than mere manipulative skills. They were given one hour for the test so as not to have to rush through it and make accidental errors.

The order of the questions has been changed here, beginning with the one missed less often so as to show the order of difficulty.

Test

Answer as many questions as you can in 60 minutes.

1. $(2a+3)(3a-2)=$ . . . .
2. The factors of $4a^2-b^2$ are . . . .
3. The factors of $p^2+14p+48$ are . . . .
4. The age 7 years ago of a man who is now X years old was . . . .
5. $(x^2+x-12)\div(x-3)=$ . . . .
6. If $4t/5-t/5-1=1-2t/5, t=$ . . . .
7. If $\frac{3}{4}x+7=28$, $x=$ . . . .
8. If $x=2$, the value of $(-2x)^3$ is . . . .
9. The square of the sum of two general numbers may be represented by . . . .
10. In the formula $A=h/2(b+b')$, the value of b, when $A=50$, $h=5$, and $b'=9$ is . . . .
11. The sum of the squares of two general numbers may be represented by . . . .
12. $-21a^6b^2/-7a^2=$ . . . .
13. The number of cents in N nickels and D dimes is . . . .
14. When $a=1$, $b=2$, and $c=0$, the value of $5a^2b^3-3ac$ is . . . .
15. In the formula $b'=2a/h-b$, if a and b do not change, when h increases the value of b' . . . .
16. Henry is X years old and his father is 5 times as old. The sum of their ages 5 years hence will be . . . .
17. $xy-7(x-y)$ minus $3xy-8(x-y)$ is . . . .
18. Will $\dfrac{2+1/a}{3}$ increase or decrease as a increases?
19. If $a+6d=5$, and $a+13d=-9$, then $a=$ . . . .
20. The factors of $2ab-1+a^2+b^2$ are . . . .
21. $a/b+c/d=$ . . . .
22. If $x=2+\sqrt{3}$, $x^2+4x+1=$ . . . .
23. The odd number next larger than the even integer 2N is . . . .
24. $x+1/x-1+x-1/1-x=$ . . . .
25. $\dfrac{x-3/7}{2}-\dfrac{x}{21/3}=$ . . . .

Ex. 1. The first question was missed by 19. The most frequent error was $6a^2—5a—6$, being an error in sign. The results showed that these 19 pupils had trouble with the middle term and lacked control of the law of signs.

Ex. 2. This was missed by 19 and omitted by 10 pupils. The typical errors were $2a—b$, failing to write but one factor; $4(a—b)\ (a+b)$; $4(a—b)^2$; and $2a+b\cdot 2a—b$. The errors shows that these pupils had no understanding of the fundamental nature of factoring. The last error, $2a+b\cdot 2a—b$, is of a different nature. The one pupil giving this error did not understand the use of the dot in showing the product of two binomials.

Ex. 3. This was missed by 19 and omitted by 12. The most common error in this example was $(p+7)^2$, being mistaken as expression for a perfect square; another common error was $p(p+14)+48$ showing a lack of any knowledge of the meaning of factoring.

Ex. 4. This was missed by 25 and omitted by 10. The three most common errors were $7—x$; $7+x$; and $7x$. These pupils did not even get the first notion of the use of general numbers—that they stand for some unknown definite number as 20 or 30.

Ex. 5. This was missed by 29 and omitted by 7. The most frequent error was a mistake in sign, the answer given being $x—4$. And a careful study of the errors made throughout the test showed great lack of control of signs.

Ex. 6. This was missed by 44 and omitted by 14. The most frequent answer was 10, due again to lack of control of signs. There were, however, 21 different answers given, and the length of this paper will not allow an analysis of the reasons for the errors.

Ex. 7. This was missed by 56 and omitted by 8. The errors resulted from lack of knowledge of the fundamental principle underlying the solution of equations.

Ex. 8. Here 68 got wrong results and one omitted it. The error most frequently made was that of the sign. Thus nearly half of those missing it gave the answer 64. Other answers were $—8x^3$, $—64x^3$, 16 and $—256$.

Ex. 9. This was missed by 68 and omitted by 12. The answers given were $(1+2)^2$; N; x^2; $\sqrt{a+b}$. Twenty-four gave

$(1+2)^2$, showing that they did not know the meaning of the term "general number."

Ex. 10. This was missed by 72 and omitted by 10. The most common answer was 1. Others were 18 1/5, 5,29, —4, and 9. By omitting the 1/2, those pupils got 1. Those getting $18\frac{1}{2}$ multiplied $(9+b)5$ and got $9+5b$.

Ex. 11. This was missed by 72 and omitted by 17. The most common answer given was 3^2+4^2. Others were N, $(x+y)^2$, $2N^2$, $\sqrt{a}+\sqrt{b}$. These errors were about the same type as those in Ex. 9 and usually made by the same pupils.

Ex. 12. This was missed by 87 and omitted by 3. This skill is so fundamental that it is hard to conceive that a pupil having had one or two years of algebra and ranking in the upper half of his class could miss this one. The most common answer was $3a^3b^2$ showing that exponents were divided instead of subtracted. Other answers were $-3a^4b^2$, $-3a^3b^2$, $3b^2$, $-3b^2$.

Ex. 13. There were 89 errors and it was omitted by 15. The most common error was ND. Other answers were N+D, N/5+D/10, and .05N+.10D. Again this exercise showed lack of knowledge of the meaning and use of general number.

Ex. 14. This was missed by 112 and omitted by 4. The most common answer was 37, found by saying $3\times1\times0=3$. This is a very common error and dates back to poor teaching of arithmetic. Another made by many was $5\times1\times8=6\times8=48$.

Ex. 15. This was either missed or omitted by 126 pupils. Of the 204 getting the correct answer there is no way of telling how many merely "guessed" right.

Ex. 16. This was missed by 119 and omitted by 20. The most common answer was $30x$. Other answers were $6x+5$; $25x$; $15x$; and $6x$. This again showed lack of knowledge of general number.

Ex. 17. This was missed by 118 and omitted by 13. There were 43 different answers due to various causes, but the outstanding error was from lack of control of the laws of signs.

Ex. 18. This was missed by 133 and omitted by 18.

Ex. 19. This was missed by 136 and omitted by 18. Had it been stated as a pair of simultaneous equations, using x and y for a and d, no doubt more would have gotten it.

Ex. 20. This was missed by 100 and omitted by 50. Some

of the answers were $(a+b)^2-1$; $(a+b)^2$; $a+b-1$; $(a-1)(a+1)+b(2a+b)$. All errors showed a failure to appreciate the nature of factoring.

Ex. 21. This was missed by 147 and omitted by 11. The common answers were $ad+bc$; $a+c/b+d$; $ad+bc/b+d$; $a+c/bd$; ac/bd; thus showing the lack of any control of this simple process.

Ex. 22. This was missed by 135 and omitted by 40.

Ex. 23. This was missed by 165 and omitted by 13. Some common answers were 2N; 3N; 5N; 3N+1; 3; $3N^2$; thus again showing lack of understanding of general numbers.

Ex. 24. This simple exercise that is taught in every school, and which is a common form, was missed by 225 and omitted by 10. There were 75 different answers. Some answers were 0; $x+1/1-x$; 2; $2x$; $2-2x$. One would never anticipate that only 95 out of 330 would get this exercise.

Ex. 25. This was missed by 179 and omitted by 81. There were 112 different answers and space does not permit me to analyze them. But only 70 out of 330 got the correct answer.

Space has not permitted me to make an adequate analysis of the causes of failure, but enough has been done to show that many pupils get but little beyond a few manipulative skills in their course in algebra, and even among these, they get a very poor control of the laws of signs. Another outstanding conclusion is that many pupils go through algebra without getting any conception of the meaning and use of general number, without which their study of algebra is of but little if any value.

A FRESHMAN PLACEMENT PROGRAM

By C. D. SMITH
Louisiana College

It should be of interest to teachers both in high school and college to follow the results of studies in placement and teaching of freshman mathematics. The results here given are of considerable significance in view of the mortality rate shown by the Year Book of the Southern Association of Colleges and Secondary Schools. The writer has been interested in this problem for some years and it is hoped that present results are merely indicative of greater progress in the near future.

Louisiana College requires all students to take freshman mathematics. Under this system the records show that for the period of eight years prior to 1928 the first course in algebra has a failing list of 39%. We found that placement in three groups did no apparent good where all are taught in the same manner. The problem seems to lie in the remedy rather than in the mere mechanics of placement. Two years ago we adopted a course consisting of a set of lessons adaptable to A, B, and C sections designed to better introduce the college mathematics and connect in a way with the advanced subjects. This should be done while we are giving the processes required for work in science by those who will take only three hours of mathematics.

Now it is not an easy matter to successfully burn the candle at both ends thereby reducing the number of failures and at the same time contributing to the success of the major students. The following brief statement will indicate the present status of the program. By placement tests we arrange the classes in three sections on the basis of scores. The scores are filed so that no influence on class marks or final grades will result. The results of placement show that practically all failures come from the lower half and none from the upper third. All first year students who made a general average above 90% in all classes were among the first twenty-five in each year's placement scores. Hence we are able to predict both the honor list and the doubtful list.

The number of failures has dropped from 39% to 33% in the first course. We shall now advise those who place very low to take a special review for three months and then follow with the freshman course. The regular C section will begin with the full set of practice reviews. The B section will take part of the review work and try a test before proceeding and the A class will take still less review practice. By revision of lessons we will be able to remove certain difficulties in learning the more difficult processes. The second year students show every indication of the ability to proceed with less difficulty than in former years. In a word the prospects are quite hopeful for greatly reducing the doubtful list and at the same time improving the foundation for later work. We hope at a future date to report the results of the present revised plan.

VOTED AT THE CLEVELAND MEETING

Whereas we, as members of The Louisiana-Mississippi Section of the Mathematical Association of America, the Louisiana-Mississippi Chapter of The National Council of Mathematics Teachers, and the Mississippi Academy of Science have been so cordially received and entertained by the city of Cleveland, the entire administrative staff, faculty and student body of Delta State Teachers College;

Whereas Dr. Westfall and Dr. Elliott have come long distances and at considerable inconvenience to themselves to appear on our programs; and,

Whereas all of those appearing on our programs have contributed much to the success of our meetings;

Therefore, be it resolved that a rising vote of thanks be extended to all those mentioned as contributing to our pleasure and profit.

The Resolutions Committee,
B. A. TUCKER, Chairman
ELIZABETH HARRIS, Secretary

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*THE AESTHETIC IN TEACHING MATHEMATICS

By W. PAUL WEBBER
Louisiana State University

The purpose of this paper is rather to emphasize and say amen to what others have suggested than to make any actual contribution to the subject.

Every art has one first principle or general major premise, viz.—the assumption that the objective aimed at is desirable. This is not borrowed from science but is humanistic. For example, the builder's art assumes that it is desirable to have buildings. Architecture assumes that beautiful imposing buildings are desirable. The art of the practice of medicine assumes that the cure of disease is desirable. The teaching art must assume that the giving of instruction is desirable. Art asserts that something is desirable or ought to be. Science asserts that

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something exists. Art proposes that something should be. Science asserting the existence of certain things proceeds to determine the conditions necessary to the creation of the proposed "should be." Art and Science join hands and determine whether the project is humanly possible.

The teaching art further assumes that excellence in instruction is desirable. The science of education must determine the conditions under which excellent instruction can be obtained. Teaching artists then attempt to do excellent teaching when the conditions are supplied. Excellence in instruction must also be defined. This in turn will depend upon the definition of education. It is not the present purpose to discuss this phase but to assume that some general agreement exists among educators on this point.

The teaching of a particular branch will have some objective peculiar to itself. There will be general objectives common to all branches. Pleasure is certainly one objective in education. Fine art and aesthetic ideas are a source of pleasure to not a few people. Hence, the artistic and aesthetic values of any branch ought to be brought out in teaching. The teacher of the branch must learn what the science of education has to say about the conditions under which the desired instruction can be realized and proceed to carry on instruction accordingly.

There has been much discussion and instruction tending to put in evidence the utilitarian values of mathematics. It may be that enough has been done in that direction to suffice for some time. It may be that other values even more delightful have been overlooked or neglectd. The contemplation of the beautiful is aesthetic. The contemplation of the beautiful is a source of pleasure and is cultural. Such ideas suggest the existence (for some at least) of an intellectual life aside from the routine of daily life. What can be derived from the study of mathematics that may contribute to the contemplation and appreciation of the beautiful? Only a few have made public their ideas along this line.

Geometry deals with forms. What forms are beautiful? What elements of form make for beauty. Some at least may be mentioned, such as,

Symmetry with respect to a line or axis.
Symmetry with respect to a center.

Pleasing proportion of parts or dimensions.

Rhythm of form or of sound.

Generally (inclusiveness) or power of a method or a formula There is a thrill to many on becoming acquainted with some general powerful method of doing something.

Precision of statement of a law or principle.

Nature abounds in the elements of beauty. Note the symmetry of form in leaves, flowers, animal bodies, distribution of internal organs in pairs. The circle, the oval the sphere and certain polygons possess symmetry. All these are obvious in nature. Nature is the place to begin the study of geometry. In nature learn the meanings of many geometric terms. This has been nicely done by Sister Alice Irene in the January Mathematics Teacher. Her article is valuable in suggestions both as to geometry and as to the kind of preparation teachers need.

Miss Gugle has elucidated another phase in her article on Dynamic Symmetry in the third year book of the National Council. This book should be in every library for the use of teachers and advanced pupils.

When the writer was a child he liked to recite the "Fives" and "Tens" of the multiplication tables. Other children did, too, There was an easy rhythm in these tables that pleased the young sense of rhythm and stimulated the memory. There is rhythm in the other tables but it is not so simple or obvious at first. Try to find the rhythm period in the "Six's" or in the $2\frac{1}{2}$'s" or the "3-1/3'rds."

There is rhythm in the binomial coefficients, well brought out in Pascal's triangle. Thus

a						1						
$a+b$					1		1					
$(a+b)^2$				1		2		1				
$(a+b)^3$			1		3		3		1			
$(a+b)^4$		1		4		6		4		1		
$(a+b)^5$	1		5		10		10		5		1	

etc.

Note also the odd fact that each line may be obtained from the line above by the addition of adjacent numbers except for the end ones which are all 1's. This scheme is striking for its simplicity, oddness and its aid to the memory. Graphs drawn with the numbers in any line as ordinates equally separated along

the x-axis will illustrate a symmetry about the central numbers.

There is a kind of rhythm in the addition formula for the sine and cosine and a sort of symmetry too. Consider

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B.$$

Consider the comprehensive precision (power) of the cosine law viz,

$$a^2=b^2+c^2-2bc\cos A$$

This law states in one line the content of three well known theorems in Geometry viz. (a) The Pythagorean Theorem when $A=90$ and $\cos A=0$; (b) The square on a side of a triangle opposite an abtuse angle etc. Here $A>90°$ and $\cos A$ is negative making the last term add as it should. (c) The same statement is true when A is acute and the last term subtracts as it should. Here is power and generality. It is noted here that $bc\cos A$ is equivalent to b times the projection of c on b to correspond to the geometric statement.

The power and simplicity of the formula in the solution of quadratic equations is forceful and inspiring. Every quadratic can be reduced to a type form. The method of solution applies to the type form and hence to all quadratics. If

$$ax^2+bx+c=0$$

$$x_1=\frac{-b+\sqrt{b^2-4ac}}{2a},\quad x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

contains the whole story of quadratics. For if $b^2-4ac=0$ the roots are equal. If $b=0$, the roots are equal with opposite signs, if $c=0$ one root is zero and the other $-b/a$, if a approaches 0, x_2 becomes infinite, and x_1 may be written

$$x_1=\left.\frac{(\sqrt{b^2-4ac}-b)(\sqrt{b^2-4ac}+b)}{2a(\sqrt{b^2-4ac}-b)}\right]=\left.\frac{b^2-4ac-b^2}{2a(\sqrt{b^2-4ac}+b)}\right]_{a=0}=-\frac{c}{b}$$

as it should be.

My experience as a teacher convinces me that not a few students get pleasure from the study of mathematics analogous to that derived from music, poetry, or dancing. Such students are by no means all men. "God bless the girl that refuses to study algebra" was not sopken by a woman.

May it not be that in our modern madness to be practical we have overlooked a world of beauty that is enticing and productive of pleasure and that will include much that is practical as a byproduct?

Higher mathematics is not less enticing in its beauty, power, and simplicity.

What could be more impressive than the fact that nearly all the laws of the physical universe are expressible in two fairly simple mathematical equations?

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*THE HARMONIC DIVISION OF A LINE

By IRBY C. NICHOLS
Louisiana State University

The object of this article is to call attention respectfully to the harmonic division of a given line, and to invite teachers of High School mathematics to do more toward acquainting their pupils with this topic than is done at present. A more serious study of it would contribute considerably toward helping the student to appreciate the significance of positive and negative directions in Algebra, of positive and negative ratios of segments of lines in Analytical Geometry, and prepare the way for a study of Synthetic Projective Geometry.

Our High School graduate usually remembers having studied article 281 of Wentworth-Smith's Plane Geometry, and he usually knows how to divide a given line AB into segments having a given positive ratio: When P, the point of division is an **internal** point. But too often he is unable to divide the same line into segments having a negative ratio: When Q, the point of division, is an **external** point.

Again our High School graduate usually knows how to bisect **internally** and **externally** the angle C of a given triangle ABC, of which AB is a given line, and, by this route, secure a harmonic division of AB, the ratio being equal to the ratio of AC to BC, the sides of the triangle ABC about the bisected angle C; but he hesitates and stops still when asked to divide AB **har-**

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monically into just any arbitrarily assigned ratio, no triangle being mentioned.

The necessary constructions are not difficult to perform, nor to understand—the subject only needs **due** and **definite** attention. A few carefully-executed constructions, with some well chosen exercises, should clear up the whole matter and put the student in a position to appreciate many of the things in Algebra and Analytical Geometry that he ordinarily misses or finds obscure. Take **line** values for m and n and divide a given line AB **internally** and **externally** into the ratio m to n, giving to m and n different relative values, of course, and taking care always to make clear the fact that the fraction m to n is **positive** when m and n are taken in the **same** directions, and **negative** when taken in the **opposite** directions.

To Divide a Given Line AB Harmonically into the Ratio m to n:

1. To divide a given line AB into segements having a positive ratio m to n, m and n being as shown in figure 1.

Construction: Take AB the given length. Draw AB′ making any convenient angle with AB. On AB′ lay off AP′ and P′B′ of lengths km and kn respectively, both being taken in the

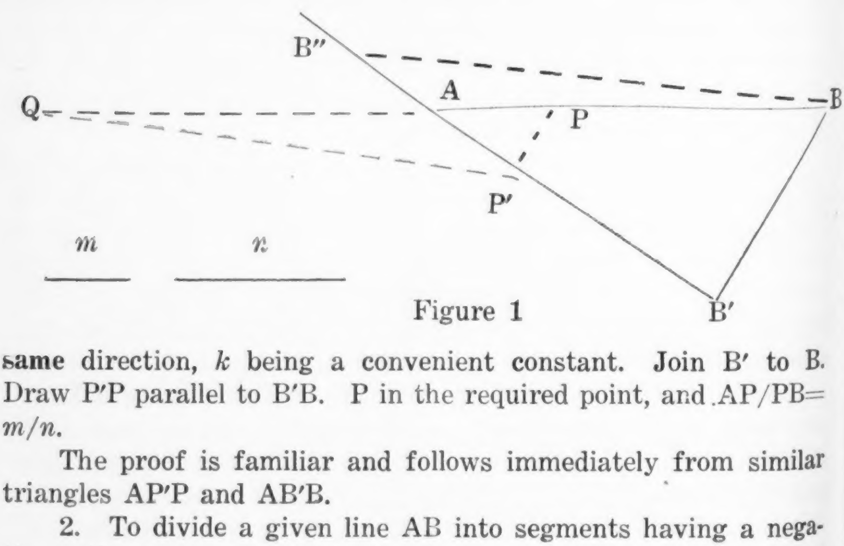

Figure 1

same direction, k being a convenient constant. Join B′ to B. Draw P′P parallel to B′B. P in the required point, and $AP/PB = m/n$.

The proof is familiar and follows immediately from similar triangles AP′P and AB′B.

2. To divide a given line AB into segments having a negative ratio.

Construction: Take AB the same as above. Take AB′ as before, or in a different position. Take AP″ equal to $k'm$ and

$P''B''$ equal to $k'n$, AP'' and $P''B''$ being taken along AB' but in **opposite** directions, k' being the same or different in value from k above. (In figure 1 shown, $k=k'$, and AB' is taken in the same direction as in case 1. Hence P' coincides with P''.) Join B'' to B, and draw through P'' (P' in Figure 1) a line $P''Q$ parallel to $B''B$. Q divides AB externally into segments having a negative ratio, and $AQ/QB=-m/n$.

Proof: From similar triangles AQP' and ABB'', $AQ/BA=AP'/B''A$; then, by composition, $AQ/(BA+AQ)=AP'/(B''A+AP')$, or $AQ/BQ=AP'/B''P'=m/n$. Reversing the order of reading BQ we have $BQ=-QB$. Hence $AQ/QB=-m/n$. Hence combining results of both cases, we have $AP/PB=AQ/BQ$, or, if read as is customary, $AP/PB=-AQ/QB$.

Incidentally, it is the custom with college folks to speak of the four collinear points A, B, P, Q, when related as just mentioned, as a **harmonic range,** and to write the range thus (ABPQ). A harmonic range obviously consists of two pairs of points, A being referred to as the conjugate of B, or B as the conjugate of A. Likewise P and Q are conjugates. It is essential to know which pairs are conjugates. Also, it is customary to assume A, B, P as given and require Q to be found. During the rest of this paper these customs will be followed.

Proceeding now with our topic, attention may be called to several familiar and important facts:

1. There is only one point Q, conjugate to P, with respect to A, B.

2. From the nature of the construction of figure 1, it is quite obvious that Q falls to the right or the left of AB according as the absolute value of m is taken greater or less than n.

3. If P be midway between A and B, Q will be at infinity.

4. If the line AB be divided harmonically at P and Q, then conversely the line PQ is divided harmonically at A and B; that is, $PA/AQ=-PB/BQ$. For $AP/PB=-AQ/QB$, by alteration, gives $AP/AQ=-PB/QB$, whence $PA/AQ=-PB/BQ$.

5. A circle on PQ as a diameter is the locus of the vertex C of a triangle ABC such that CP and CQ are the internal and

external bisectors respectively of the angles at C. Said circle is called the circle of Apollonius.

Almost all High School graduates seem to be under the impression that the locus of C is a straight line.

In order to disabuse the mind of the High School student of the impression that the harmonic range is, of necessity, concerned primarily with the bisection of angles, several exercises involving harmonic ranges should be included in his course in Geometry. The following are suggested samples:

Exercise 1. To find Q, the conjugate of P, given the collinear points A, B, P of (ABPQ).

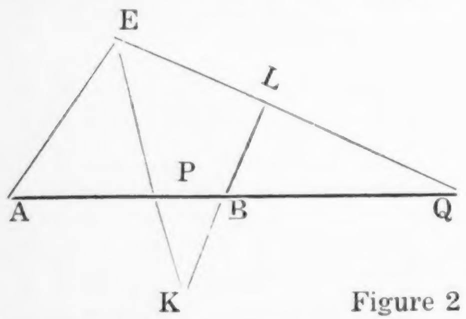

Figure 2

Construction: A, B, P are given as shown in figure 2. AE and KL, any pair of parallel lines, are drawn through A and B respectively. Through P, any line is is drawn cutting AE at E and KL at K. BL is taken equal to KB. EL is drawn and produced until it cuts AB extended at Q. Q is the harmonic conjugate of P. The construction obviously admits of only one solution. If P be an **internal** point with reference to AB, Q will be an **external** point as shown; if P be an **external** point with reference to A and B, Q will be an internal point, and certain of the steps in the construction of figure 2 will be reversed.

Proof: From similar triangles, AEP and BKP, AP/PB= AE/KB=AE/BL=AQ/BQ=—AQ /QB. Therefore AP/PB=— AQ/QB and Q is the harmonic conjugate of P in (ABPQ).

Exercise 2. If AB be made the diameter of a circle (O) and a chord be drawn perpendicular to AB at P, the chord cutting the circle at T and T′, then the tangent to the circle at T or T′ will cut AB extended at Q, the harmonic conjugate of P with reference to A, B.

Proof: From figure 3, it is clear that we shall have the two similar right triangles OPT and OTQ from which OP/OT= OT/OQ. Whence OP/OB=OB/OQ, since OT=OB.

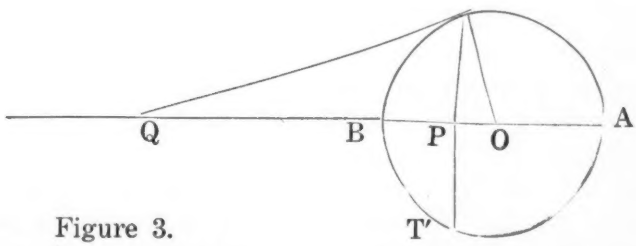

Figure 3.

Then $(OB+OP)/(OB-OP)=(OQ+OB)/(OQ-OB)$, or $(AO+OP)/(OB-OP)=(OQ+AO)/(OQ-OB)$, or $AP/PB=AQ/BQ=-AQ/QB$, which shows that Q is the harmonic conjugate of P with respect to A, B.

If P be an external point with respect to A, B, then Q will be an internal point, and may be found by reversing the order of the last steps of our construction.

A teacher, wishing several numerical illustrations for class room use, may use this general theorem to good advantage by giving numerical values to AB, the diameter of the circle, and to OP, the distances of the chord TP from the center O. For instance, if $AB=26$ and $OP=5$, then we will find, in order, OQ, PQ, AP, PB, AQ and QB respectively equal to 33.8; 28.8, 18, 8, 46.8, -20.8. $AP/PB=18/8=9/4$, and $-AQ/QB=-46.8/-20.8=9/4$. Therefore $AP/AQ=-AQ/QB$, and A, B, P, Q is a harmonic range.

In passing, it may be stated without proving, that, given any two circles cutting each other at right angles, a line passing through the center of either of them cuts them in four points of range.

Exercise 3. A third very interesting illustration of a harmonic range is that of the four notable points of Euler's line: the orthocenter. H, the circumcenter O, the centroid G, and the center N of the nine-point circle of the triangle ABC.

For it is well known that these four points are collinear and

Figure 4.

that G and N are respectively 1/3 and 1/2 of the length of OH from O, and hence $OG/GN=-OH/HN$ for $OG/GN=2/1$ and also $-OH/HN=-6/-3=2/1=$, thus showing that G and H

divide the line ON harmonically. Or GN/NH=—GO/OH, for GN/NH=1/3, and also —GO/OH=—2/—6=1/3, thus showing also that GH is divided harmonically by O and N. Here, too, is an illustration of fact 4 on page 17 above.

Exercise 4. If a triangle ABC be drawn on the given line AB as a base, and P, a point dividing AB internally, be joined to C, then AS and BS through S, any point of CP, determine M and N on AC and BC respectively, so that a line through M N will cut AB extended in Q, the harmonic conjugate of P.

For since the lines AN, BM, CP drawn from the vertices of the triangle ABC are concurrent at S, we have by Ceva's theorem[1].

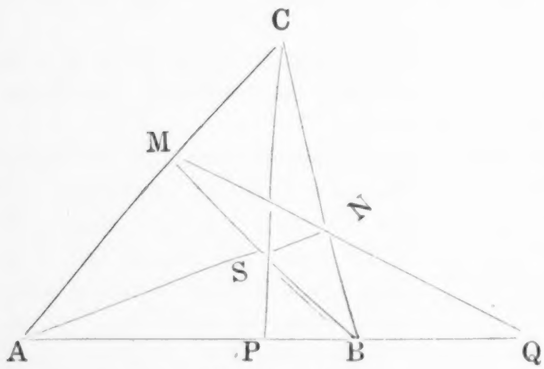

Figure 5.

AP/PB·BN/NC·CM/MA=1; and since the transversal MNQ cuts the sides of the triangle ABC in MNQ, we have by the Theorem of Menelaus[2].

AQ/QB·BN/NC·CM/MA=—1.

From these two equations, by division and re-arrangement, AP/PB=—AQ/QB, proving that A, B, P, Q is a harmonic range.

Other versions of this theorem are given in Projective Geometry in the study of complete quadrangles and complete quadrilaterals.

[Note 1. "The lines joining the vertices of a given triangle to a given point determine on the sides of the triangle six segments such that the product of three of these segments having no common end is equal to the product of the remaining three segments."

Note 2. "The six segments determined by a transversal on the sides of a triangle are such that the product of three non-consecutive segments is equal to the product of the three others."]

Exercise 5. A fifth and final illustration is this:

Given the incircle touching the sides B C, C A, A B of the △ABC in the points X, Y, Z respectively; given Y Z produced to meet B C in T; then B, C, T, X is a harmonic range.

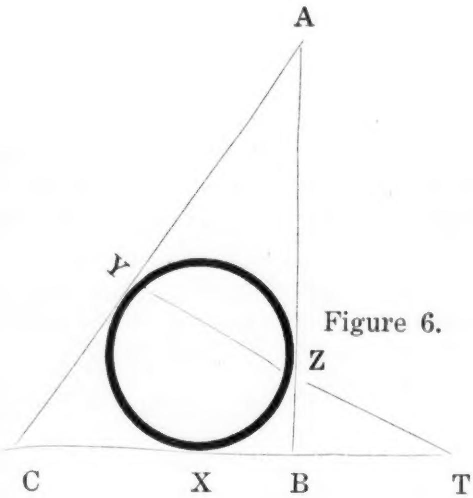

Figure 6.

Proof: From the Theorem of Menelans, $AZ/ZB \cdot BT/TC \cdot CY/YA = -1$. Noting that $AZ = YA$, $CY = XC$, and $ZB = BX$ we have, after re-arranging, $BX/XC = -BT/TC$. Therefore T and X divide BC harmonically and B, C, T, X forms a harmonic range.

COMMENT ON RECENT BOOKS

By DORA M. FORNO

Corrective Arithmetic, Vol. II. By Wirth J. Osburn.
Publishers: Houghton Mifflin Co., New York, 1929.

Readers of Corrective Arithmetic, Vol. I, who are familiar with the construction of this work, will welcome with great interest this new publication.

The major emphasis in the volume is put upon things that have proved their value, based upon an extensive research program. The function of Corrective Arithmetic is (a) to provide a method of diagnosing pupils' needs and (b) to provide suitable experiences in the way of practice material for the relief of those needs.

A very interesting chapter on problem-solving stresses the point that we can increase the operation of transfer by teaching the likenesses and differences that are involved by means of cues which are listed.

Common fractions, decimals, denominate numbers, and ratio and proportion are considered with the purpose of presenting means of relieving the present trouble in their teaching. Arithmetic as a liberal art and as a preparation for higher mathematics are discussed with the purpose of showing how the upper

grade arithmetic is enriched by the introduction of topics of business, commercial and industrial; problems without numbers, graphs; mensuration; the use of literal quantities; etc.

The chapter on "What to Teach in Each Grade" emphasizes some very significant facts—(1) the need for objective experiences, (2) making an inventory of what pupils know upon entering the grade, and (3) suggested cues for teaching certain facts or processes.

S. T. SANDERS IN ACCOUNT WITH MATHEMATICS NEWS LETTER OVER PERIOD FROM MARCH, 1929 TO MARCH, 1930

EXPENSES

*Printing April (1929) News Letter	$ 55.20
Printing September (1929) News Letter	51.60
Printing October (1929) News Letter	53.80
Printing November (1929) News Letter	55.40
Printing December (1929) News Letter	61.05
Cost of stamped wrappers	19.06
TOTAL	$296.11

RECEIPTS

Total amount of donations from institutions, corporations, such donations not being intended as mere subscriptions	$ 61.00
Total amount of donations from individuals, such donations not intended as mere subscriptions	$ 68.00
New subscriptions and renewals of subscriptions since the Lafayette meeting, March, 1929	114.96
Personal contribution	9.00
TOTAL	$252.96

Present debt (exclusive of present issue) due Ortlieb Press only ....$ 43.15

*Vouchers for printing cost payments are in the hands of Secretary-treasurer Dale.

SOME OPINIONS OF THE NEWS LETTER

Chicago, Ill., December 16, 1929.

To Chairman Hardin,
Shreveport, La.
My dear Professor Hardin:

I have your letter of recent date telling of the plans and work of the Louisiana-Mississippi Section of the Mathematical Association of America. I rejoce in every successful step taken by this Section, not only as representative of the collegiate insti-

tutions in the two states but also, and quite as important, as a cooperative body with the Mississippi-Louisiana Branch of the National Council of Teachers of Mathematics in the secondary field. I am quite proud of the fact that I was present and assited when this affiliation between these two organizations took place and I have ever since watched with the greatest interest this cooperation between the collegiate and secondary teachers of mathematics in Louisiana and Mississippi. It was, I think, the first example of such cooperation, but since that time several other instances of such affiliation have followed your example.

One of the most forward-looking activities of your combined organizations is the publication of the "Mathematics News Letter." This, also, is a pioneer and has attracted wide attention. It is a very essential element in the promotion of professional interest among both secondary and college teachers of mathematics in the two states. The American Mathematical Monthly and the Mathemaics Teacher are **national** organs, the one in the collegiate field and the other in the secondary field, but they cannot supply the need of a **local** organ such as the News Letter.

I would like to commend to every administrative officer, college or secondary, in Louisiana and Mississippi the worthwhileness of the "Mathematics News Letter." I venture the assertion that it has already proved its worth in the stimulation of better teaching and the development of professional consciousness, but I am sure that it has hardly begun to show what its possibilities are. Any institution in the two states which, may contribute to its support will be making an investment that will yield manifold results in the promotion of higher standards of teaching in the department of mathematics.

Too much cannot be said for the devotion and optimism of the men and women who by their labor and their cash contributions have made possible the present development of this project, but the burden should now be more widely distributed and shared by all individuals and institutions who are willing to participate in such worthy service for the cause of mathematics in that part of the country and indirectly for the betterment of teaching in all departments by the example thus afforded by mathematics.

Yours very sincerely,

H. E. SLAUGHT.

Wheaton College, Wheaton, Ill.

The Mathematics News Letter,
Baton Rouge, Louisiana

My dear sirs:

Enclosed will find check for one dollar as payment for subscription to your News Letter. I have a copy of the January number. Could I have a copy of the missing numbers for current year?

Do you have the issues for past years bound in volumes for sale? If so, please send me some information about prices for same.

I am interested in your journal and would appreciate having all back numbers from date of publication. Sincerely,

CHESTER R. HILLARD.

Pubs. of Mathematics News Letter,
Care of Louisiana State University,
Baton Rouge, La.

Gentlemen:

Recently we have had calls for your magazine, Mathematics News Letter, but in carefully checking our records we do not find that we have ever received any rates from you.

The Moore-Cottrell Agency is the oldest as well as one of the largest subscription agencies in the United States, having agents located all over the world. We are purely a wholesale magazine agency, receiving our business from over 30,000 dealers, therefore will you not fill out and return to us the enclosed sheet giving us your regular as well as your best net price for subscriptions to Mathematics News Letter?

Kindly mark and return to the writer so, there will be no delay in it's reaching the proper department. Very truly yours,

J. W. LEWIS.

For the satisfaction of, and in justice to, all paid-up subscribers, it should be explained that the present "Jan.-Feb.-March" issue of the Mathematics News Letter is NOT to be taken as three separate issues, composed into one, but rather as one of the eight regular issues. The sixth, seventh and eighth issues of Volume 4 will be issued as the April, May and June numbers.